Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 1 (Group)
香港数学竞赛 (2002 – 2003)
决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 n，k 皆为自然数，且 $1<k<n$。若 $\frac{(1+2+3+\cdots+n)-k}{n-1}=10$ 及 $n+k=a$，求 a 的值。

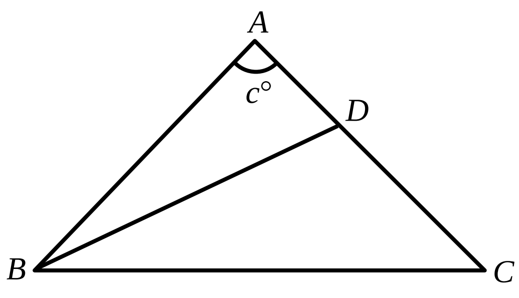
Given that n and k are natural numbers and $1<k<n$. If $\frac{(1+2+3+\cdots+n)-k}{n-1}=10$ and $n+k=a$, find the value of a.

2. 已知 $(x-1)^2+y^2=4$，其中 x 和 y 是实数。若 $2x+y^2$ 的极大值是 b，求 b 的值。

Given that $(x-1)^2+y^2=4$, where x and y are real numbers. If the maximum value of $2x+y^2$ is b, find the value of b.

3. 如图一，ΔABC 是一个等腰三角形，其中 $AB=AC$。若 $\angle B$ 的角平分线交 AC 于 D 且 $BC=BD+AD$。设 $\angle A=c^\circ$，求 c 的值。

In Figure 1, ΔABC is an isosceles triangle and $AB=AC$. Suppose the angle bisector of $\angle B$ meets AC at D and $BC=BD+AD$. Let $\angle A=c^\circ$, find the value of c.



图一
Figure 1

4. 两质数之和为 105。若这两质数之积为 d ，求 d 的值。

Given that the sum of two prime numbers is 105. If the product of these prime numbers is d, find the value of d.

Hong Kong Mathematics Olympiad (2002 – 2003)

Final Event 2 (Group)

香港数学竞赛 **(2002 – 2003)**

决赛项目 **2 (**团体**)**

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1. 设方程 $ax(x+1)+bx(x+2)+c(x+1)(x+2)=0$ 有根 1 和 2。若 $a+b+c=2$，求 a 的值。

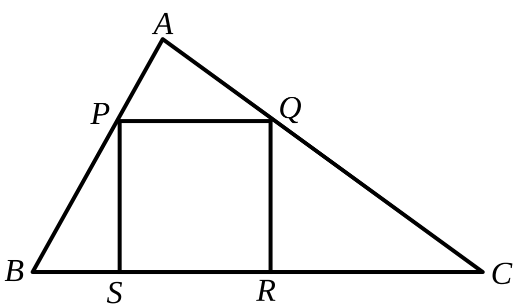
Given that the equation $ax(x+1)+bx(x+2)+c(x+1)(x+2)=0$ has roots 1 and 2. If $a+b+c=2$, find the value of a.

2. 设 $48^x=2$，$48^y=3$。若 $8^{\frac{x+y}{1-x-y}}=b$，求 b 的值。

Given that $48^x=2$ and $48^y=3$. If $8^{\frac{x+y}{1-x-y}}=b$, find the value of b.

3. 如图一，正方形 $PQRS$ 内接于 ΔABC。ΔAPQ、ΔPBS 和 ΔQRC 的面积分别为 4、4 和 12。若正方形 $PQRS$ 的面积为 c，求 c 的值。

In Figure 1, the square $PQRS$ is inscribed in ΔABC. The areas of ΔAPQ, ΔPBS and ΔQRC are 4, 4 and 12 respectively. If the area of the square $PQRS$ is c, find the value of c.



图一

Figure 1

4. 在 ΔABC 中，$\cos A=\dfrac{4}{5}$ 和 $\cos B=\dfrac{7}{25}$。若 $\cos C=d$，求 d 的值。

In ΔABC, $\cos A=\dfrac{4}{5}$ and $\cos B=\dfrac{7}{25}$. If $\cos C=d$, find the value of d.

Hong Kong Mathematics Olympiad (2002 – 2003)

Final Event 3 (Group)

香港数学竞赛 (2002 – 2003)

决赛项目 3 (团体)

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1. 设 f 为一函数，$f(1)=1$，并对任意整数 m 及 n，$f(m+n)=f(m)+f(n)+mn$。若 $a=\dfrac{f(2003)}{6}$，求 a 的值。

Let f be a function such that $f(1)=1$ and for any integers m and n, $f(m+n)=f(m)+f(n)+mn$. If $a=\dfrac{f(2003)}{6}$, find the value of a.

2. 若 $x^{\frac{1}{2}}+x^{-\frac{1}{2}}=3$，$b=\dfrac{x^{\frac{3}{2}}+x^{-\frac{3}{2}}-3}{x^2+x^{-2}-2}$，求 b 的值。

Suppose $x^{\frac{1}{2}}+x^{-\frac{1}{2}}=3$ and $b=\dfrac{x^{\frac{3}{2}}+x^{-\frac{3}{2}}-3}{x^2+x^{-2}-2}$, find the value of b.

3. 已知 $f(n)=\sin\dfrac{n\pi}{4}$，其中 n 是整数。若 $c=f(1)+f(2)+\cdots+f(2003)$，求 c 的值。

Given that $f(n)=\sin\dfrac{n\pi}{4}$, where n is an integer. If $c=f(1)+f(2)+\cdots+f(2003)$, find the value of c.

4. 已知函数 $f(x)=\begin{cases}-2x+1, & 當\ x<1 \\ x^2-2x, & 當\ x\geq 1\end{cases}$。若 d 是 $f(x)=3$ 的最大整数解，求 d 的值。

Given that $f(x)=\begin{cases}-2x+1, & \text{when } x<1 \\ x^2-2x, & \text{when } x\geq 1\end{cases}$. If d is the maximum integral solution of $f(x)=3$, find the value of d.

Hong Kong Mathematics Olympiad (2002 – 2003)

Final Event 4 (Group)

香港数学竞赛 **(2002 – 2003)**

决赛项目 **4 (**团体**)**

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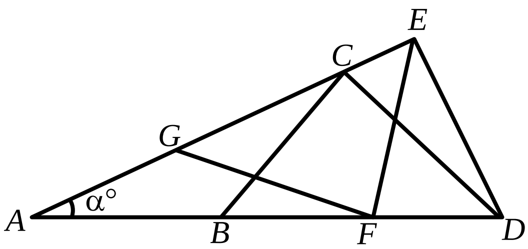
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如图一， AE、AD 是直线且 $AB = BC = CD = DE = EF = FG = GA$ 。若 $\angle DAE = \alpha°$，求 α 的值。

   In Figure 1 , AE and AD are two straight lines and $AB = BC = CD = DE = EF = FG = GA$. If $\angle DAE = \alpha°$, find the value of α .



图一

Figure 1

2. 设 $P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_8x^8$ 为八次多项式，其中 a_0 , a_1 , … , a_8 为实数。若 $P(k) = \frac{1}{k}$ 当 $k = 1 , 2 , \ldots , 9$，及 $b = P(10)$，求 b 的值。

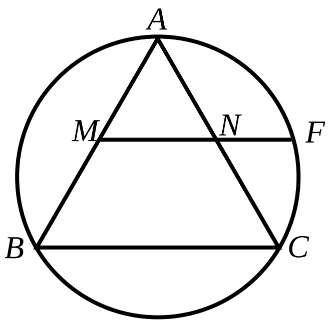
   Suppose $P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_8x^8$ is a polynomial of degree 8 with real coefficients a_0 , a_1 , … , a_8 . If $P(k) = \frac{1}{k}$ when $k = 1 , 2 , \ldots , 9$, and $b = P(10)$, find the value of b .

3. 已知 x，y 为两正整数使 $xy - (x + y) = \mathrm{HCF}(x, y) + \mathrm{LCM}(x, y)$，其中 $\mathrm{HCF}(x, y)$ 和 $\mathrm{LCM}(x, y)$ 分别是 x 和 y 的最大公因子和最小公倍数。若 c 是 $x + y$ 的最大可能的值，求 c。

   Given two positive integers x and y , $xy - (x + y) = \mathrm{HCF}(x, y) + \mathrm{LCM}(x, y)$, where $\mathrm{HCF}(x, y)$ and $\mathrm{LCM}(x, y)$ are respectively the greatest common divisor and least common multiple of x and y . If c is the maximum possible value of $x + y$, find c .

4. 如图二，ΔABC 是等边三角形，M 及 N 分别是 AB 及 AC 的中点，F 是直线 MN 与圆 ABC 的交点。若 $d=\dfrac{MF}{MN}$，求 d 的值。

In Figure 2 , ΔABC is an equilateral triangle , points M and N are the midpoints of sides AB and AC respectively, and F is the intersection of line MN with the circle ABC. If $d=\dfrac{MF}{MN}$, find the value of d .



图二

Figure 2